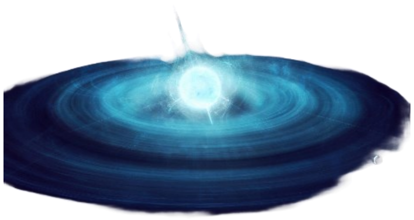
Modified from PBS (https://www.pbs.org/deepspace/classroom/activity4.html)



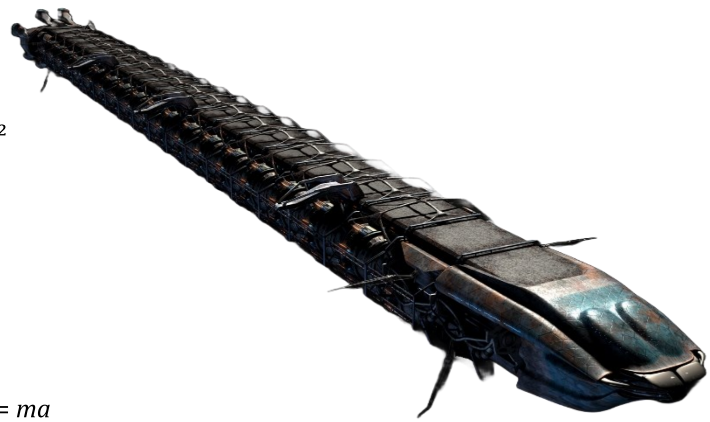
Objectives:

- Review previously learned physics concepts
- Apply relativity to motion near a blackhole

Procedure:

It is the year 2070 and you and your crew of interstellar astronauts are prepared to take the first journey of humans beyond Pluto! Your mission is to test, firsthand, the predictions of Einstein's General Theory of Relativity. Specifically, your goal is a black hole some 10,000 light-years away. Because of its distance, the trip will be a multigenerational one. Your offspring, and theirs, and so on, will be trained in your task to complete the mission.

As chief engineer, you have been assigned to ensure the safety of your crew and craft. You are armed with your trusty Interstellar Navigational Handbook from a college course you took on interstellar travel. In it you find the following constants and equations:



Constants

- Speed of light: $c = 3.00\times10^{8}$ m/s
- Gravitational constant: $G = 6.67\times10^{-11}$ Nm^2/kg^2
- Mass of Earth: $M_E = 5.98\times10^{24}$ kg
- Radius of Earth: $R_E = 6.37\times10^{6}$ m
- Acceleration of Earth's gravity: $g = 9.81$ m/s^2

Equations

- 1 light-year = 9.46×10^{15} m
- 1 m^3 = 1×10^{6} cm^3
- density = m/V $d = vt$ $v = at$ $F = ma$
- Volume of a sphere: $V = \frac{4}{3}\pi r^3$, where r = radius
- Centripetal Force: $F_C = \frac{mv^2}{r}$, where m = mass, v = velocity, r = radius of motion
- Period of object in circular motion: $T = \frac{2\pi r}{v}$, where v = velocity, r = radius
- Gravitational Force: $F_G = \frac{GmM}{r^2}$, where m = mass of smaller object, M = mass of larger object, r = distance between centers of objects
- Escape Velocity: $v_{esc} = \sqrt{\frac{2GM}{r}}$, where M = mass of larger object, r = distance between centers of objects

Your spacecraft, *Veracious*, is a Lockheed Martin X-120 Far Range Prober. Its mass, including cargo and robot probes, is 10,000 kg. Each robot probe weighs 100 kg, has a height of 10 m, and has a thrust capacity of 50,000 N of thrust for a duration of 10 hours.

The *Veracious* has a maximum controllable thrust of 800,000 N and uses the newest waste/debris fusion-reactor system as its propellant system (it uses waste materials and cosmic dust in fusion reactions to produce energy). This system requires minimal on-board fuel. (Engineer's Note: The mass of the *Veracious* will remain constant throughout the trip, unlike earlier conventional spacecraft whose mass decreased as fuel was used up.)

What makes the Far Ranger Prober really special, though, is its quark fusion quantum accelerator, which has the capability of boosting the *Veracious*' velocity to 80 percent of the speed of light (or reducing it by the same amount). This ability is crucial because even at that speed, the trip will take you more than ten thousand years. Relativistic effects at such high velocities will, however, make the trip shorter for the astronauts on board the *Veracious*, if not for the Earth observers back home.

As chief engineer, the success of the mission rests firmly in your hands (and head!). Good luck!!

Questions:

1. Your energy-calculation notes from college show that for any object to escape the gravitational pull of a planet, star, and so on, the object must first achieve escape velocity, or v_{esc}. What velocity must the *Veracious* achieve to escape Earth?
2. To determine how long you must power the thrusters, you must first determine the acceleration they are capable of generating for the *Veracious*. Use the above information to do this calculation.
3. You know that an acceleration of more than 10 *g*'s is fatal to humans, so quickly you calculate how many *g*'s the above acceleration is. Are the *g*-forces at full thrust survivable?
4. How long must the engines be powered at full thrust to achieve escape velocity?
5. Once the *Veracious* is clear of the Earth's gravity, NASA operating procedure calls for the firing of the quark fusion quantum accelerator to boost velocity to 80 percent of the speed of light. After that, no acceleration by either fusion reactor will operate except for course corrections and the return trip to Earth. At the rate of 0.8c then, how many years will it take for you to reach your destination as measured by the control facility on Earth?
6. Einstein determined that according to his Special Theory of Relativity, time actually slows for those traveling at speeds near the speed of light. Using the formula for time dilation and the time calculated as seen from Earth in the previous calculation, determine how many years will pass as observed by the occupants of the *Veracious*.
7. Along the way, you discover a neutron star. This neutron star is typical, having a mass of 2.0×10^{30} kg and a radius of 10,000 m. Although neutron stars are incredibly hot (1,000,000 K) they emit relatively little visible electromagnetic radiation, which explains why you did not observe this star before. The crew decides to take advantage of the unanticipated opportunity to explore this cousin to the black hole. The stellar astronomers wish to send a robot probe (able to withstand incredible temperatures) to the surface to obtain a 1 kg sample of the star.

   Getting the robot probe to the surface would be straightforward, they explain—release the probe into a spiraling orbit until it finally nears the surface of the neutron star. When close enough, a scoop would reach down for a sample as the probe continues to orbit just above the star. Probe thrusters would then be used to return the probe to the *Veracious*.

   In fitting the robot probe with a sample scoop, you wonder what volume of sample is needed for a 1 kg sample.
   a. First determine the density of the neutron star. For reference, recall that water has a density of 1×10^3 kg/m^3 and that the Earth has an average density of 5.52×10^3 kg/m^3.
   b. What volume will a 1 kg sample of the star take up in m^3? In cm^3? Will you need a shovel, a spoon or an ultraminiature scoop?
8. For any object to maintain an orbit about another, the centripetal force, F_C, must be provided by the gravitational force, F_G; thus $F_C = F_G$. If the robot probe were to orbit right at the surface of the star, what velocity must it maintain?
9. At that velocity, how long would it take the robot probe to circle the star?
10. Calculate the VERTICAL escape velocity required to leave from the surface of the star.
11. If the robot probes are equipped with enough fuel to provide 50,000 N of thrust for 10 hours, would they be able to escape the neutron star? (Hint: Calculate the maximum acceleration of robot probe first.)
12. As the chief engineer, what do you think of the proposed plan to retrieve a sample from the neutron star? Explain.
13. Many generations after the *Veracious* first left Earth, one of the stellar astronomers makes a peculiar observation. Five approaching stars have exactly the same spectra, implying that they are made of exactly the same kinds and amounts of elements. Even more curious is the fact that the five stars are positioned exactly at the corners of a perfect pentagon.

   Upon hearing this news, the relativistic physicist of the crew gets very excited. "What we're observing," he explains, "is a gravitational lensing effect caused by a black hole! You see very intense gravitational fields that have the ability to bend light rays. What we are actually seeing is a single star on the far side of a black hole. Normally we would not be able to see the light at all because the black hole is in the way. However, the gravity of the black hole is bending that light around the black hole and directing it to us, where we see it as five identical stars!"

"We can't see the black hole itself, of course, because no light escapes it to reach our eyes. Black holes are only detectable by the gravitational effects we can observe on other objects. If a black hole occurs as half of a binary system, detection visibly is more obvious."

As the *Veracious* approaches the black hole, the crew is able to calculate its mass, your distance from it, and the degree to which it tugs on your ship. The crew finds that the mass of the black hole (2.2×10^{30} kg) is surprisingly similar to that of the neutron star's! However, the black hole's physical radius is even smaller, at 3 km.

You know better than to send a robot probe to the surface of the black hole because you realize that you could never retrieve it. Just to confirm the issue, however, calculate the escape velocity necessary to leave the surface of this black hole. Explain why it is not only impractical to send and retrieve a probe from the surface, it is IMPOSSIBLE!

14. The black hole is only the size of a small town, yet its gravitational influence stretches well beyond its physical boundaries. Given the mass of the black hole the mass of the object of interest and one's distance from the black hole one can easily compute the gravitational force, F_G, experienced by an object.
    a. The robot probes are 10 meters in height and are vertically divided into three basic sections—the bottom portion is the lander, the midsection is the scientific and general controller, and the top is for navigation and communication with a remote pilot. Which portion would experience the greatest attraction to the black hole? Which the least? Why?
    b. To examine the difference of the gravitational force between top and bottom, first derive a generalized formula for such a difference in force (assume that the top section of the probe will always be 10 m farther away from the black hole than the bottom is).What is the difference in gravitational force from top to bottom if the craft's bottom is 1.0×10^{8} m from the center of the black hole.
    c. Because the lower portion of the craft is being pulled more forcefully than the top, the DIFFERENCE in these two forces has to be compensated for by the craft's hull. The greater the difference in the forces, the more apt the craft is to be torn apart. Any astronaut falling into a black hole would experience the same stretching and then be quickly ripped apart. This phenomenon is referred to as a tidal effect and is exactly the same principle in action as that for the ocean tides that occur on Earth. The hull of the robot probe is capable of withstanding 1000 N of force before being ripped apart. Would the probe be destroyed at the altitude given in (a) and (b)? Explain.
    d. Determine how close the probe can get to the hole without being destroyed. Make a polynomial equation and solve it on technology.

The crew launches a probe to the distance from the black hole that you specified. The robot safely returns to the *Veracious*, and as the results of your study are radioed back to Earth, the crew celebrates the completion of a mission thousands of years in the making. In 10,000 years, the celebration will continue back on Earth when your data is received.

As a last experiment while orbiting the black hole, the crew sends a robot probe directly into the hole. As it approaches the event horizon, the boundary beyond which there is no return, the robot appears to turn redder and redder in color. Additionally, the radio signals of data that it sends back to the *Veracious* exhibit longer and longer wavelengths. The relativity physicist explains that this redshift is not completely explained by the fact that the probe is moving away from the *Veracious*. Most of the stretching of the wavelengths occurs because the robot and radio signals are experiencing a slowing of time! Analogous to the slowing of time from movement at high velocities is a slowing of time that occurs for objects in intense gravitational fields (like near black holes). The time of the robot relative to that of the *Veracious* is slowing as it approaches the hole. The frequencies of light the probe is emitting appear to the crew of the *Veracious* as lower in frequency and longer in wavelength, hence the reddening color.

Although as viewed from the *Veracious*, the probe appears redder and seems to talk a lower and lower pitch, the probe notices nothing peculiar about its own behavior. It does, however, listen to the signals the *Veracious* is sending it and finds that those signals are blue-shifted. As it gets closer and closer to the black hole, the probe sees that it is falling ever faster into the hole. If tidal forces don't rip the probe apart first, the probe would, according to the robot's clock, within minutes fall into the black hole to be lost forever.

To the crew of the *Veracious*, the probe's color gets redder and redder as well as dimmer and dimmer. Finally, the probe would appear black as the light from the probe, like all light from a blackhole, is stretched in wavelength to black. Interestingly, if the crew members could see the probe reach the edge of the black hole, they would see the probe stop and never proceed into the blackhole. This is because at this point, the probe's time is infinitely short relative to the *Veracious*—time has effectively stopped.

Such is the strange world of Einstein's General Theory of Relativity.

6. $\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$